Non-Parametric Estimation of Correlation Functions

Brincker, Rune; Rytter, Anders; Krenk, Steen

Publication date:
1991

Document Version
Publisher's PDF, also known as Version of record

INSTITUTTET FOR BYGNINGSTEKNIK
DEPT. OF BUILDING TECHNOLOGY AND STRUCTURAL ENGINEERING
AALBORG UNIVERSITETSCENTER • AUC • AALBORG • DANMARK

FRACTURE & DYNAMICS
PAPER NO. 31

Submitted to Experimental Mechanics

R. BRINCKER, A. RYTTER & S. KRENK
NON-PARAMETRIC ESTIMATION OF CORRELATION FUNCTIONS

JUNE 1991 ISSN 0902-7513 R9120

The FRACTURE AND DYNAMICS papers are issued for early dissemination of research results from the Structural Fracture and Dynamics Group at the Department of Building Technology and Structural Engineering, University of Aalborg. These papers are generally submitted to scientific meetings, conferences or journals and should therefore not be widely distributed. Whenever possible reference should be given to the final publications (proceedings, journals, etc.) and not to the Fracture and Dynamics papers.

INSTITUTTET FOR BYGNINGSTEKNIK
DEPT. OF BUILDING TECHNOLOGY AND STRUCTURAL ENGINEERING
AALBORG UNIVERSITETSCENTER • AUC • AALBORG • DANMARK

FRACTURE & DYNAMICS
PAPER NO. 31

Submitted to Experimental Mechanics

R. BRINCKER, A. RYTTER & S. KRENK
NON-PARAMETRIC ESTIMATION OF CORRELATION FUNCTIONS
JUNE 1991 **ISSN 0902-7513 R9120**

NON-PARAMETRIC ESTIMATION OF CORRELATION FUNCTIONS

Rune Brincker
Associate Professor of Civil Engineering
University of Aalborg, Denmark

Anders Rytter
Research Assistant in Experimental Mechanics
Rambøll & Hannemann, Denmark

Steen Krenk
Professor of Computational Mechanics
University of Aalborg, Denmark

Abstract

In this paper three methods of non-parametric correlation function estimation are reviewed and evaluated: the direct method, estimation by the Fast Fourier Transform and finally estimation by the Random Decrement technique. The basic ideas of the techniques are reviewed, sources of bias are pointed out, and methods to prevent bias are presented. The techniques are evaluated by comparing their speed and accuracy on the simple case of estimating autocorrelation functions for the response of a single degree-of-freedom system loaded by white noise.

1. Introduction

Measurements from dynamic systems usually consists of a large amount of data. For instance data from an on-line surveying system may consist of multichannel measurements closely sampled over long periods of time. Typically the information in the data is reduced by unknown random loading and noise.

Sometimes it is desirable to use all the information available in the time series and possible to handle the large amount of data in the system identification proces. In this case the best choice is to use an identification technique where a parametric model is fitted directly to the original time series, Ljung [1], Söderström and Stoica [2] , Pandit and Wu [3].

In other cases one would like to reduce the original time series to small amounts af data

describing the most important proporties of the time series. The basic idea is to minimize the influence of noise and random loading by averaging characteristic functions like spectral density functions or correlation functions. The advantage is that significantly smaller amounts of data has to be tranferred and stored, and that the system identification proces becomes much faster because the models are fitted to a significantly smaller number of data points.

There is a price to pay for this data compression however: information is lost, and bias might be introduced.

Information will be lost in a data compression proces, because it is not possible to contain all the detailed information hidden in the time series in correlation functions or spectral density functions. Therefore, system parameters estimated from such "interface" functions, will always show larger variance than parameters estimated by effective fitting of models directly to the time series as mentioned above. In practice however, the choice is governed by a trade off between accuracy and speed, and sometimes it is beneficial to accept a small incease in variance for a large decrease in the time used in the estimation process.

From an engineering point of view however, it is also important to prevent systematic errors - in estimation terminology: bias. In cases where proporties of data are represented in compressed form, it is important that the estimated "interface" functions are unbiased, i.e. that they contain correct information about system parameters.

It is well known, that spectral density functions for non-periodic signals estimated by Fourier transform techniques will always will be biased, and that bias in the frequency domain can be reduced, but not prevented or removed. However, the bias problems are much easier to handle in the time domain, in fact it is possible to prevent bias in the time domain, i.e. by estimation of correlation functions. This is the major advantage of representing the proporties of the time series by correlation functions instead of spectral density functions.

The aim of this paper is to give a simple and self-contained presentation of three known techniques for correlation function estimation: the direct method where the correlation function is estimated by direct calculation of the correlation integral, the FFT technique, where the correlation function is estimated by the Fast Fourier Transform, and finally the Random Decrement technique, where the correlation function is estimated by simple averaging. The basic ideas of the three techniques are reviewed, sources of bias are pointed out, and methods to prevent the bias problems are presented.

Finally the three techniques are evaluated by comparing the speed and the accuracy estimating the auto correlation function for a single-degree-of-freedom system loaded by white noise. The system response is simulated using an ARMA (2,1) model which forms an exact solution in the discrete time space.

2. The Direct Method

The direct method was applied for estimation of correlation functions up to the mid sixties where the modern methods of Fast Fourier Transfomation were introduced.

The algorithm is simple and easy to use and programme, but slow in the most cases. For short correlation function estimates however, it is only a bit slower than the unbiased Fast Fourier Transform technique.

For the stationary stochastic processes $X(t), Y(t)$ the correlation function $R_{XY}(\tau)$ is defined by, Papoulos [3], Bendat and Piersol [6]

$$R_{XY}(\tau) = E[X(t+\tau)Y(t)] \tag{1}$$

In practice however, it is not possible to perform ensemble averaging. Instead the processes are assumed to be ergodic, and the ensemble averaging is replaced by time averaging on the realizations $x(t), y(t)$.

$$R_{XY}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_0^T x(t+\tau)y(t)dt \tag{2}$$

Real time series are always limited however, they are known over some finite time domain, say $[0;T]$. To keep the arguments inside the definition set, for positive time lags the upper integration limit cannot exceed $T-\tau$, and the natural modfication of eq. (2) is then

$$\hat{R}_{XY}(\tau) = \frac{1}{T-\tau} \int_0^{T-\tau} x(t+\tau)y(t)dt \ , \quad \tau \in [0;T] \tag{3}$$

forming a correlation function estimate. The corresponding result for negative time lags are found by modifying the lower integration limit, or using that $R_{XY}(-\tau) = R_{YX}(\tau)$. The estimate given by eq. (3) is unbiased since the expectation of the estimator is equal to the definition given by eq. (1)

$$E[\hat{R}_{XY}(\tau)] = \frac{1}{T-\tau} \int_0^{T-\tau} E[X(t+\tau)Y(t)]dt = R_{XY}(\tau) \tag{4}$$

The corresponding biased estimate is defined by

$$\hat{R}^w_{XY}(\tau) = \frac{1}{T} \int_0^{T-\tau} x(t+\tau)y(t)dt \ , \quad \tau \in [0;T] \tag{5}$$

It is seen that the estimates are related by $\hat{R}^w_{XY}(\tau) = w(\tau)\hat{R}_{XY}(\tau)$, where $w(\tau)$ is a triangular window, the so-called "basic lag window", given by $w(\tau) = \frac{T-\tau}{T}$, Newland [8].

3. The Fast Fourier Transforms Method

The Fast Fourier Transform is an effective algorithm for calculation of Fourier coefficients, Brigham [5]. The basic ideas were discovered in the forties by Danielson and Lanczos, [9], but the technique became known by the work of Cole and Tukey, [10] and implemented in larger scale from the the mid-sixties.

Application of the Fast Fourier Transform (FFT) technique is based on the periodic estimate

$$\hat{R}^p_{XY}(\tau) = \frac{1}{T}\int_0^T x(t+\tau)y(t)dt \tag{6}$$

where the data segment pair $x(t), y(t)$ originally defined on $[0;T]$ is made periodic by setting $x(t+T) = x(t)$, $y(t+T) = y(t)$. Using the results from appendix A, it is easy to see that the periodic estimate can be written as a convolution

$$\hat{R}^p_{XY}(\tau) = x(\tau) * y(-\tau) \tag{7}$$

and that $\hat{R}^p_{XY}(\tau)$ is periodic with period T. Therefore the length of the of the periodic estimate is only T and not $2T$ as for the direct estimates introduced in the preceeding section. Let $y(t)$ and Y_n be a Fourier Transform pair, $y(t) \leftrightarrow Y_n$, then $y(-t) \leftrightarrow \overline{Y}_n$ where the overbar denotes complex conjugation, se eq. (A.3) in appendix A, and the Fourier Transform S_n of $\hat{R}^p_{XY}(\tau)$ is therefore given by

$$\hat{R}^p_{XY}(\tau) \leftrightarrow X_n\overline{Y}_n = S_n \tag{8}$$

where $x(t) \leftrightarrow X_n$. This equation, together with the FFT algorithm, form the basis of correlation function estimation by the Fast Fourier Transform.

The idea is to take the time series, divide them into smaller data segment pairs of the length T, estimate the spectral values S_n for all the data segment pairs, average over the pairs to reduce random uncertainty and finally transform back to the time domain to obtain the correlation function estimate by inverse FFT.

The advantage is, that this technique - because of the efficiency of the FFT algorithm - is faster than the direct method. The problem is that the estimate is biased.

The bias is introduced in the calculation of the correlation integral, eq. (6), by the assumption of periodicity of the data segments. If the argument $t+\tau$ is larger than T,

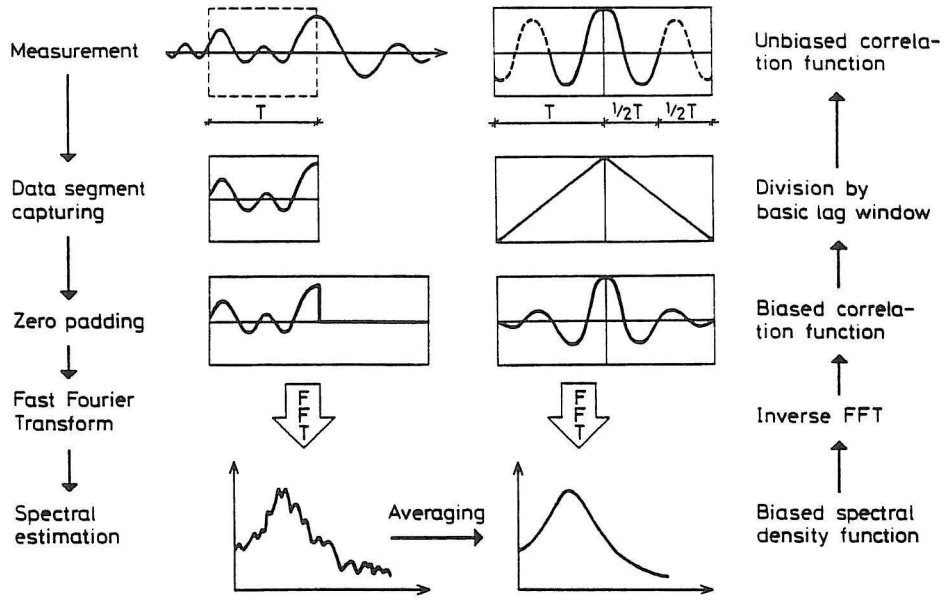

Figure 1. Unbiased correlation function estimation by FFT.

then the factor $x(t+\tau)$ in the correlation integral is replaced by $x(t+\tau-T)$, and the ends of the periodic estimate of the correlation function is polluted by "wrap around bias".

The bias might be large or small depending on how well the assumption of periodicity is satisfied, but once the bias is there, it cannot be removed. The bias might be reduced in the frequency domain by introduction of suitable windows, Brigham [5].

In the time domain however, the bias can be prevented by padding zeroes, Bendat and Piersol [6]. From the original data segment pair $x(t), y(t)$ defined on $[0;T]$ a new pair is defined

$$[x_0(t), y_0(t)] = \begin{cases} [x(t), y(t)] & t \in [0;T] \\ 0 & t \in]T;2T] \end{cases} \tag{9}$$

This completely removes the wrap around bias on the periodic estimate. The reason is that the added zeroes make sure that the integrand in the convolution integral vanishes outside the original definition set $[0;T]$. Thus the zero padding doubles the length of the correlation function estimate, and makes it correspond exactly to the biased estimate $\hat{R}^w_{XY}(\tau)$.

The effect of padding zeroes is therefore that the more or less arbitrary wrap around bias is replaced by the well defined window bias. As it appears from the previous section, the

window bias is simply removed by division by the basic lag window $w(\tau) = (T - \tau)/T$.

This technique of unbiased correlation function estimation by FFT is illustrated in figure 1. Since the variance increases at the ends of the estimate due to decreasing information, it might be considered to discard the outermost parts of the correlation function estimate. In figure 1 the possibility of discarding the last half part of the estimate is indicated.

4. The Random Decrement Method

The Random Decrement (RDD) technique is a fast technique for estimation of correlation functions for Gaussian processes by simple averaging.

The RDD technique was developed at NASA in the late sixties and early seventies by Henry Cole and co-workers [11-14], just a little later than the development of the FFT technique.

The method has not achieved wide spread use as the FFT technique even though one of the avantages is speed. In fact the technique is much faster than the FFT technique in many cases.

The basic idea of the technique is to estimate a co-called RDD signature. If the time series $x(t)$, $y(t)$ are given, then the RDD signature estimate $\hat{D}_{XY}(\tau)$ is formed by averaging N segments of the time series $x(t)$

$$\hat{D}_{XY}(\tau) = \frac{1}{N}\sum_{i=1}^{N} x(\tau + t_i)|\, C_{y(t_i)} \tag{10}$$

where the time series $y(t)$ at the times t_i satisfies the trig condition $C_{y(t_i)}$, and N is the number of trig points. The trig condition might be for instance that $y(t_i) = a$ (the level crossing condition) or some similar condition. The algorithm is illustrated in figure 2. In eq. (10) a cross signature is estimated since the accumulated average calculation and the trig condition are applied to two different time series. If instead the trig condition is applied to the same time series as the data segments are taken from, an auto signature is estimated.

However, one of the problems of the technique are that the theoretical basis has been unclear and is still being discussed. Most of old references on the RDD Technique, including the original papers by Cole and co-workers argue on a more or less heuristic basis that the RDD signature formed by averaging time series segments from the output of a stochasic loaded system should describe system proporties only.

This interpretation was modified by Vandiver et al, [15], who defined the RDD auto signature as the conditional expectation $D_{XX}(\tau) = E[X(\tau)|\, X(0) = a]$, and proved that in the case of the level crossing trig condition applied to a Gaussian process, the RDD signature is simply proportional to the auto correlation function

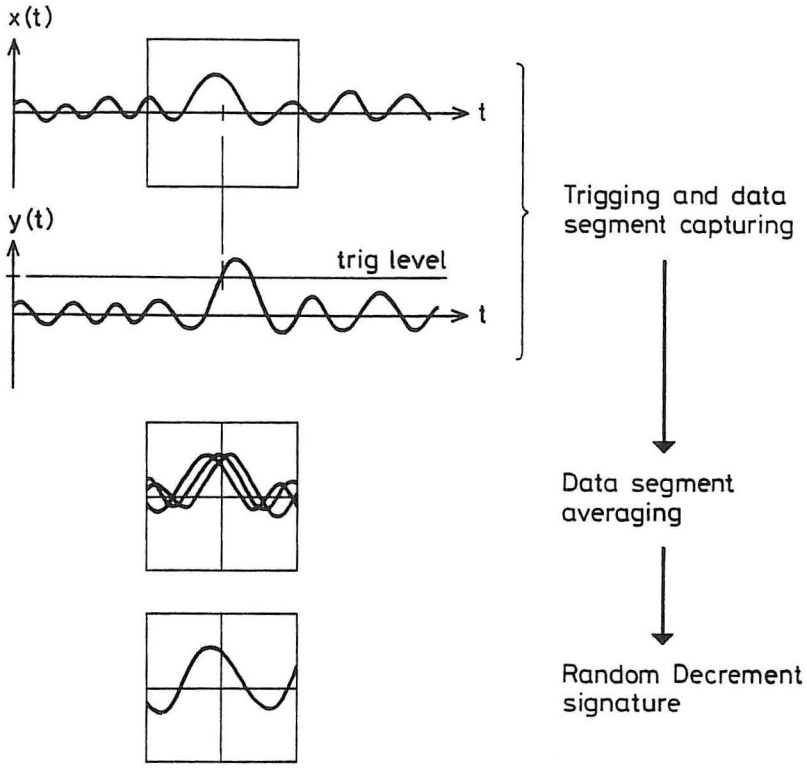

Figure 2. Determination of the Random Decrement signature.

$$D_{XX}(\tau) = E[X(\tau)|\, X(0) = a] = \frac{R_{XX}(\tau)}{\sigma_X^2} a \tag{11}$$

where a is the trig level and σ_X^2 is the variance of the proces. The original interpretation of the RDD signature is therefore correct only under the asumption of white noise loading where the free decay is proportional to the correlation function.

It is not difficult to generalize Vandivers result to the cross signature case. Let $X(t)$ and $Y(t)$ be stationary, zero mean, Gausian processes. The dependency between the processes are then completely described by the covariance matrix

$$Cov[\begin{bmatrix} X(\tau) \\ \\ Y(0) \end{bmatrix} [X(\tau)\; Y(0)]\,] = \begin{bmatrix} \sigma_X^2 & R_{XY}(\tau) \\ \\ R_{XY}(\tau) & \sigma_Y^2 \end{bmatrix} \tag{12}$$

Now, using the definition of the RDD signature as a conditional expectation and using eq.(B.4) in appendix B, taking the Gaussian vectors $\underline{X}$ and $\underline{Y}$ as scalars equal to $X(\tau)$ and $Y(0)$ respectively, the generalization of eq. (11) is easily obtained

$$D_{XY}(\tau) = E[X(t)|\, Y(0) = a] = \frac{R_{XY}(\tau)}{\sigma_Y^2} a \tag{13}$$

One of the qualities of the RDD technique is that the RDD estimate is "born unbiased". This is easily verified

$$E[\hat{D}_{XY}(\tau)] = \frac{1}{N} \sum_{i=1}^{N} E[X(\tau + t)|\, Y(t) = a] = D_{XY}(\tau) \tag{14}$$

The result given by eq. (13) is limited to the case of the level trig condition. It is possible however, to derive a kind of fundamental solution, forming a basis for application of different trig conditions. Conditioning on both Y and $\dot{Y}$ defining the Gaussian vectors $\underline{X} = X(\tau)$ and $\underline{Y}^T = [Y(0)\ \dot{Y}(0)] = [a\ v]$ and using eq. (B.4) in appendix B we obtain, Krenk and Brincker [16]

$$D_{XY}(\tau) = E[X(\tau)|\, Y(0) = a, \dot{Y}(0) = v] = \frac{R_{XY}(\tau)}{\sigma_Y^2} a - \frac{R'_{XY}(\tau)}{\sigma_{\dot{Y}}^2} v \tag{15}$$

where $R'_{XY}(\tau)$ is the derivative of the correlation function and where $\sigma_{\dot{Y}}^2$ is the variance of the derivative proces $\dot{Y}(t)$, for a Gaussian proces given by $\sigma_{\dot{Y}}^2 = -R''_{YY}(0)$. From this fundamental solution it is possible to explain the meaning of the RDD signature for several trig condition of practical interest

$$\begin{array}{lrcl} A: & Y(0) = a & \Rightarrow & D_{XY}(\tau) \propto R_{XY}(\tau) \\ B: & \dot{Y}(0) = v & \Rightarrow & D_{XY}(\tau) \propto R'_{XY}(\tau) \\ C: & \dot{Y}(0) = 0,\ Y(0) > a & \Rightarrow & D_{XY}(\tau) \propto R_{XY}(\tau) \\ D: & Y(0) = 0,\ \dot{Y}(0) > v & \Rightarrow & D_{XY}(\tau) \propto R'_{XY}(\tau) \end{array} \tag{16}$$

The result for trig condition A is found using that the distribution of $\dot{Y}(t)$ is symmetrical and independent of $Y(t)$. The last term in eq. (15) will therefore vanish, and the result becomes proportional to the correlation function in agreement with eq. (13). The result for condition B is obtained by a similar argument, and the results for the conditions C and D follows directly from eq. (15).

An estimate of the variance on the RDD signature in closed form might be found by obtianing the conditional variance using eq. (B.5) and assuming the averaged data segments to be independend, Krenk and Brincker [16].

Even though the RDD estimates are "born" unbiased as shown above, the implemetation might introduce bias. At least two sources of bias are known at present, but if these bias errors are prevented, the remaining bias errors will be extremely small at least for ligthly damped systems. The two bias sources are "trig point sorting" bias and "trig window" bias, both illustrated and discussed in Brincker et al [17].

"Trig point sorting" bias means bias introduced by non-representative data segments, typically obtained by sorting the trig points. The bias is prevented by using time series containing a large number of trig points, and by using all the trig points in the time series.

The "trig window" bias is introduced by improper implementation of the trig condition. A sampled time series is only known at certain times t_i. This means, that it is not possible for instance to obtain the exact time t the realization $y(t_i)$ crosses through the level a, and therefore the theoretical level crossing condition cannot be realized on sampled data. The condition must be modified by introduction of a finite size trig window. Vertical, horizontal and general slanted trig windows might be used, and it is possible to show, that the bias as a first approximation will give rise only to a time shift of the RDD estimate, Krenk and Brincker [16]. The bias can be removed by shifting the signature back to the rigth origin, or prevented by using self-compensating trig windows.

Some of the important elements of the theory of the RDD technique, the fundamental solution, closed form solutions for the variance on the estimate and bias introduced by different kind of windows are developed in Krenk and Brincker [16], and applications are illustrated in Brincker et al [17], [18].

5. Speed and accuracy

In this section the three techniques are evaluated comparing the speed and accuracy for estimation of autocorrelation functions. The autocorrelation functions are estimated on the output from a single degree-of-freedom (SDOF) system loaded by white noise.

The output from the SDOF system is simulated by an ARMA (2,1) model

$$x_n = \Phi_1 x_{n-1} + \Phi_2 x_{n-2} + a_n - \Theta a_{n-1} \tag{17}$$

where n is the discrete time $t_n = n\Delta t$, Φ_1, Φ_2 is the Auto Regressive (AR) parameters, Θ is the Moving Average (MA) parameter and a_m is a time series of independent Gaussian distributed numbers with zero mean. If the ARMA parameters are chosen as, Pandit and Wu, [3], section 7.3

$$\Phi_1 = 2\exp(-\zeta\omega_0\Delta t)\,\cos(\omega_d\Delta t) \tag{18.a}$$

$$\Phi_2 = -\exp(-2\zeta\omega_0\Delta t) \tag{18.b}$$

$$\Theta = -P \pm \sqrt{P^2-1}\,;\ |\Theta| < 1 \tag{18.c}$$

where

$$P = \frac{\omega_d \sinh(2\zeta\omega_0\Delta t) - \zeta\omega_0 \sin(2\omega_d\Delta t)}{2\zeta\omega_0 \sin(\omega_d\Delta t)\cosh(\zeta\omega_0\Delta t) - 2\omega_d \sinh(\zeta\omega_0\Delta t)\cos(\omega_d\Delta t)}$$

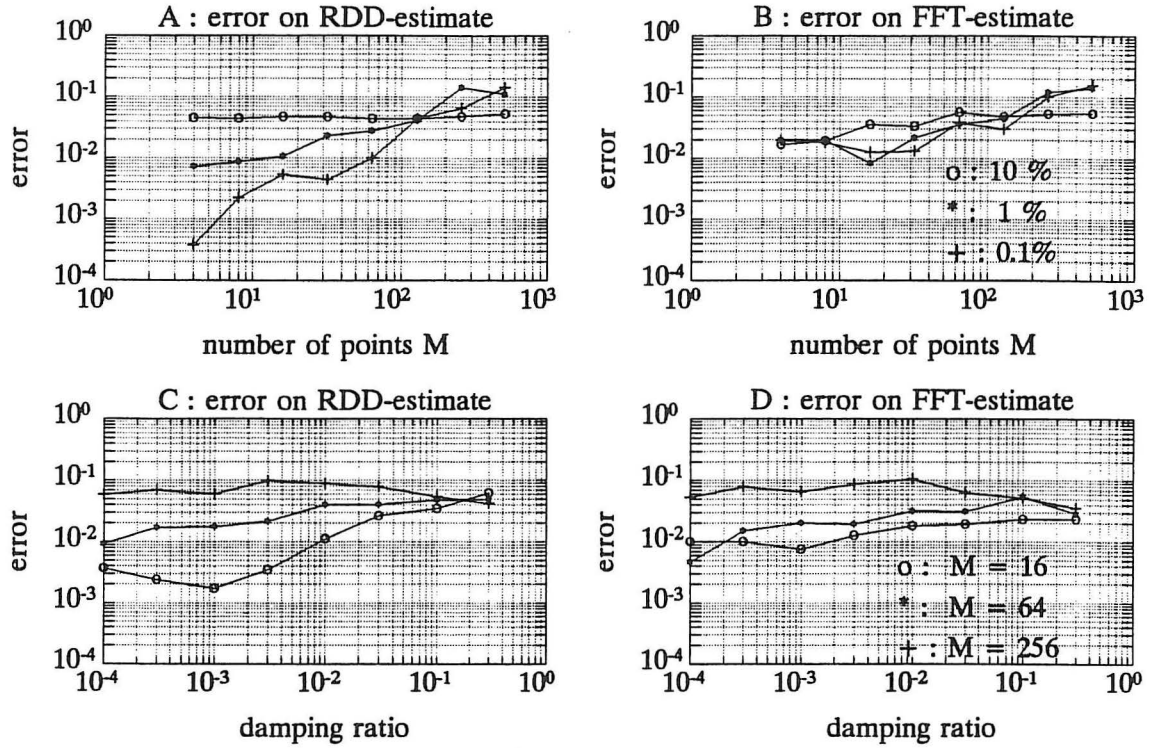

Figure 3. Accuracy of autocorrelation function estimation by the Random Decrement technique (RDD), fig 3a and 3c and by the Fast Fourier Transform (FFT) technique, 3b and 3d.

where ω_0 is the undamped natural angular frequency, ζ is the damping ratio and ω_d is the damped natural frequency of the SDOF system $\omega_d = \omega_0\sqrt{1-\zeta^2}$, then the ARMA model given by eq. (17) is the representation of the continuous SDOF system in the discrete time space, and it can be shown, that the discrete autocorrelation function of the time series x_n given by eq. (17) is exactly equal to the sampled autocorrelation function of the corresponding continuous process. The autocorrelation function of the simulated proces is therefore known to be, Crandall and Mark [7]

$$R_{XX}(t_m) = \exp(-\zeta\omega_0 t_m)\left(\cos(\omega_d t_m) + \frac{\zeta\omega_0}{\omega_d}\sin(\omega_d t_m)\right); \quad t_m \geq 0 \qquad (19)$$

and if an estimate of the correlation function $\hat{R}_{XX}(t_m)$ is obtained from the simulated time series, the estimation error ϵ can be calculated directly

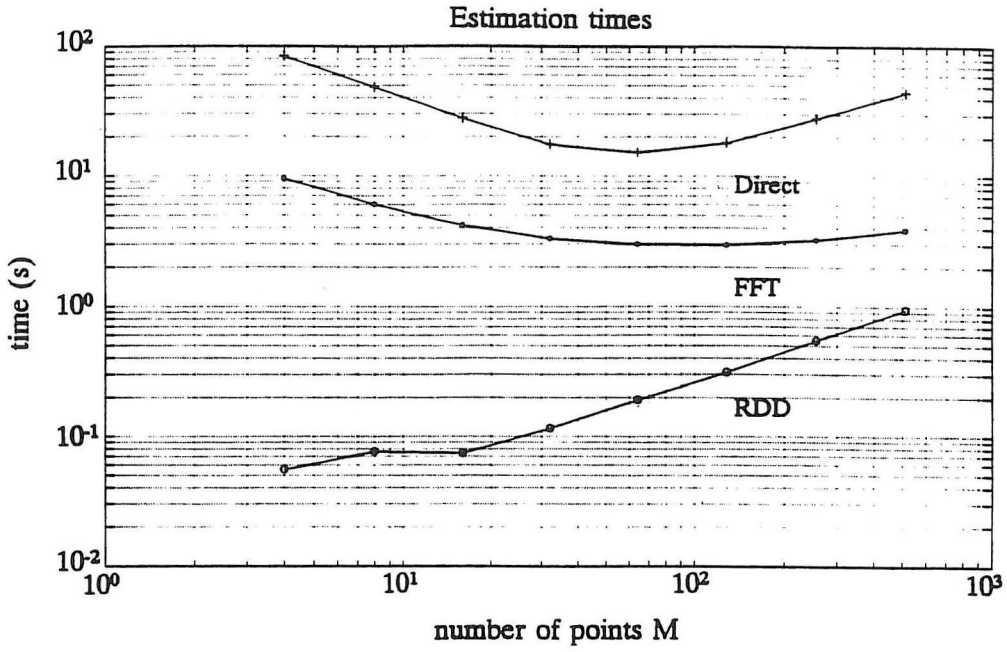

Figure 4. Times for calculation of autocorrelation function estimates by the Random Decrement technique (RDD), the Fast Fourier Transform (FFT) technique and the direct technique for a time series of 4000 data points using a 33 MHz 386 PC with 387 co-processor.

$$\epsilon^2 = \frac{1}{M\sigma_X^2} \sum_{m=0}^{M} (R_{XX}(m\Delta t) - \hat{R}_{XX}(m\Delta t))^2 \tag{20}$$

where M is the number of points in the auto correlation estimate.

All the simulations were performed modelling a system with a period of 0.5 seconds corresponding to $\omega_0 = 12.57$ rad/s, the time spacing between sample points was 0.051 seconds and the length of all the time series was 4000 points. The simulations were performed using the PC version of the MATLAB software package, [17], except the algorithm for estimation of the RDD signature which was programmed in the C programming language and linked to the MATLAB software by the MATLAB user function interface. All calculations were made on a 33 MHz 386-based PC with a 387 mathematical co-processor.

Only unbiased versions of the techniques were used as explained in the preceeding sections, and the RDD technique was used with the trig condition C in eq. (16) and $a = \sigma_X$.

Figure 3 illustrates the accuracy of the three techniques. Each point in the figure is the average of five values each estimated from time series of 4000 points. The direct technique and the FFT will yield the same estimates within the round-off accuracy, and therefore

the results for the FFT technique also represent the results for the direct technique.

In figures 3a and 3b the estimation error is shown as a function of the number of points M in the estimate for three different damping ratios, heavy damping $\zeta = 0.1$, moderate damping $\zeta = 0.01$, and light damping $\zeta = 0.001$. It appears from the results that for heavy damping and moderate damping the FFT and RDD techniques have about the same estimation errors, but that for light damping and short estimates, the estimation error for the RDD estimates is significantly smaller than the estimation error for the FFT estimates.

The same tendency can be seen also in the results shown in figure 3.c and 3.d. Here the estimation error is shown as a function of the damping ratio for three different lengths, short estimates $M = 16$, medium length estimates $M = 64$ and long estimates $M = 256$. The estimation errors for the short and the medium lenghts estimates are about the same for the two techniques, but for the short estimates, the estimation error for the RDD estimates is significantly smaller than the estimation error for the FFT estimates in the low damping region $\zeta < 0.01$

Figure 4 shows the observed estimation times for the three techniques as a function of the number of points M in the estimates. As it appears from the results, the direct technique is always the slowest and the RDD technique is always the fastest of the three. This migth change however for extremely long estimates, $M > 512$, but for short estimates the RDD technique is significantly faster than the two others. For $M \leq 64$ there is an average speed difference at about a factor of 100 betweem the FFT and RDD technique.

However, the speed results reflects not only the efficiency of the techniques, but also the way the computer treats numbers. The results reflects the ratio between computation time for addition (RDD estimation) and multiplication (FFT estimation and direct estimation). This ratio is different from computer to computer, and therefore the results given in figure 4 are not universal. The results are an indication - a strong indication - that in the most cases the RDD technique will be the most efficient technique to use for calculation of short correlation function estimates.

6. Conclusions

Three techniques are known to day for non-parametric estimation of correlation functions: the direct technique, the Fourier Transform technique and the Random Decrement technique.

All three techniques can be implemented to yield unbiased correlation function estimates, but there are differences in efficiency and accuracy.

The results in this investigation show that if correlation functions are estimated on output from systems with heavy or moderate damping then the the overall accuracy of the techniques are about the same. If correlation function estimates are short and the correlation functions are estimated on output from systems with ligth damping, then the

accuracy of the Random Decrement technique is better than for the two others.

The results in this investigation show - in agreement with common knowledge - that the direct technique is the slowest of the three, and the results indicate, that for calcution of short estimates, in the most cases the Random Decrement technique will be significantly faster than the Fast Fourier Transform.

Acknowledgements

Financial support from the Danish Technical Research Council and the Danish Energy Agency is gratefully acknowledged.

References

[1] Ljung, Lennart: "System Identification - Theory for the User". Prentice-Hall, Inc., 1987.

[2] Söderström, T. and P. Stoica: "System Identification". Prentice Hall, 1987.

[3] Pandit, S.M. and S. WU: "Time Series and System Analysis with Applications". John Wiley and Sons, 1983.

[4] Papoulis, Athanasios: "Probability, Random Variables, and Stochastic Processes". McGraw-Hill, 1984.

[5] Brigham, E.O.: "The Fast Fourier Transform". Prentice-Hall, Inc., 1974.

[6] Bendat, J.S. and A.G. Piersol: "Random Data - Analysis and Measurement Procedures". John Wiley & Sons, 1986.

[7] Crandall, S.H. and W.D. Mark: "Random Vibrations in Mechanical Systems". Academic Press, 1973

[8] Newland, D.E.: "An Introduction to Random Vibration and Spectral Analysis", Second Edition, Longman Group Limited, 1984.

[9] Danielson, G.C. and C. Lanczos: "Some Improvements in Practical Fourier Analysis and Their Application to X-ray Scattering From Liquids". J. Franklin Inst., Vol. 233, 1942.

[10] Cooley, J.W. and J.W. Tukey: "An Algorithm for the Machine Calculation of Complex Fourier Series". Mathematics of Computation, Vol. 19, April 1965.

[11] Cole, A. Henry: "On-the-line Analysis of Random vibrations". AIAA/ASME 9th Structures, Structural and Materials Conference, Palm Springs, California, April 1-3, 1968.

[12] Cole, A. Henry: "Failure Detection of a Space Shuttle Wing Flutter by Random Decrement". NASA, TMX-62,041, May 1971.

[13] Cole, A. Henry: "On-line Failure Detection and damping Measurement of Space Structures by Random Decrement Signatures". NASA, CR-2205, March 1973.

[14] Chang, C.S.: "Study of Dynamic Characteristics of Aerodynamic Systems Utilizing Randomdec Signatures". NASA, CR-132563, Febr. 1975.

[15] Vandiver, J.K., A.B. Dunwoody, R.B. Campbell and M.F. Cook: "A Mathematical Basis for the Random Decrement Vibration Signature Analysis Technique". J. Mechanical Design, Vol. 104, April 1982.

[16] Krenk, Steen and Rune Brincker: "Theory of the Random Decrement Technique." Submitted to Journal of Sound and Vibration.

[17] Brincker, Rune, J.L. Jensen and Steen Krenk: "Spectral Estimation by the Random Dec Technique." Proc. 9th Int. conf. on Experimental Mechanics, Copenhagen August 20-24, 1990.

[18] Brincker, Rune, Steen Krenk and J.L. Jensen: "Estimation of Correlation Functions by the Random Decrement Technique" Proc. 9th Int. Modal Analysis Conference (IMAC), Firenze, Italy, April 14-18, 1991.

[19] PC-MATLAB for MS-DOS Personal Computers, The Math Works, Inc., 1989.

Appendicis

APPENDIX A. Fourier transforms of finite length data

The Fourier series of a function $f(t)$ defined on $[0;T]$ is defined by

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{in\omega t} \ , \ \omega = \frac{2\pi}{T} \tag{A.1}$$

Using the ortogonality relation

$$\int_0^T e^{i(n-m)\omega t} dt = T\delta_{nm} \tag{A.2}$$

where δ_{nm} is the Kronecker delta, the Fourier coefficients or the Fourier transform of $f(t)$ is obtained

$$F_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega t} dt \tag{A.3}$$

The functions $f(t)$ and F_n is said to form a Fourier transform pair

$$f(t) \leftrightarrow F_n \tag{A.4}$$

Since the base functions $e^{in\omega t}$ are all defined on $]-\infty;\infty[$ it is natural to use eq. (A.1) to extend the definition set for $f(t)$ to $]-\infty;\infty[$. In that case $f(t)$ becomes periodic with period T. Now let $g(t)$ and $h(t)$ be periodic functions with period T. If the convolution is defined as

$$g(t) * h(t) = \frac{1}{T}\int_0^T g(t-\tau)h(\tau)d\tau \tag{A.5}$$

then by using eq. (A.2) it is easy to show that

$$g(t) * h(t) \leftrightarrow G_n H_n \tag{A.6}$$

The corresponding theorem for convolution in the frequency domain

$$G_n * H_n = \sum_{k=-\infty}^{\infty} G_{n-k} H_k \tag{A.7}$$

is found to

$$g(t)h(t) \leftrightarrow G_n * H_n \tag{A.8}$$

APPENDIX B. Condition on Gaussian variables via regression

Let $\underline{X}$ and $\underline{Y}$ be Gaussian vectors, i.e. the elements $X_1, X_2, ...X_n; Y_1, Y_2, ...Y_m$ are jointly normal distributed and are therefore completely described by the expectation and the covariance

$$E\begin{bmatrix}\underline{X}\\ \\ \underline{Y}\end{bmatrix} = \begin{bmatrix}\underline{\mu}_X\\ \\ \underline{\mu}_Y\end{bmatrix} \; ; \; Cov[\begin{bmatrix}\underline{X}\\ \\ \underline{Y}\end{bmatrix}[\underline{X}^T \; \underline{Y}^T]] = \begin{bmatrix}\underline{\underline{C}}_{XX} & \underline{\underline{C}}_{XY}\\ \\ \underline{\underline{C}}_{YX} & \underline{\underline{C}}_{YY}\end{bmatrix} \tag{B.1}$$

We now define the vector

$$\underline{Z} = \underline{X} - \underline{\underline{B}}\,\underline{Y} \tag{B.2}$$

Since all the variables are Gaussian, $\underline{Z}$ will be independent of $\underline{Y}$ if and only if

$$\begin{aligned} Cov[\underline{Z} \; \underline{Y}^T] &= \underline{\underline{0}} \\ &= \underline{\underline{C}}_{XY} - \underline{\underline{B}}\,\underline{\underline{C}}_{YY} \\ &\Downarrow \\ \underline{\underline{B}} &= \underline{\underline{C}}_{XY}\,\underline{\underline{C}}_{YY}^{-1} \end{aligned} \tag{B.3}$$

and the conditional expectation and covariance is obtained using that $\underline{Z}$ is independent of $\underline{Y}$

$$\begin{aligned} \underline{\mu}_{X|Y} &= E[\underline{X}|\,\underline{Y}] \\ &= E[\underline{Z} + \underline{\underline{B}}\,\underline{Y}|\,\underline{Y} = \underline{y}] \\ &= E[\underline{Z}] + \underline{\underline{B}}\,\underline{y} \\ &= \underline{\mu}_X + \underline{\underline{B}}(\underline{y} - \underline{\mu}_Y) \end{aligned} \tag{B.4}$$

$$\begin{aligned} \underline{\underline{C}}_{XX|Y} &= Cov[\underline{X} \; \underline{X}^T|\,Y] \\ &= Cov[(\underline{Z} + \underline{\underline{B}}\,\underline{Y})\,(\underline{Z} + \underline{\underline{B}}\,\underline{Y})^T|\,\underline{Y}] \\ &= Cov[\underline{Z} \; \underline{Z}^T|\,\underline{Y}] \\ &= Cov[\underline{Z} \; \underline{Z}^T] \\ &= \underline{\underline{C}}_{XX} - \underline{\underline{B}}\,\underline{\underline{C}}_{YX} \end{aligned} \tag{B.5}$$

FRACTURE AND DYNAMICS PAPERS

PAPER NO. 3: J. D. Sørensen: *PSSGP: Program for Simulation of Stationary Gaussian Processes.* ISSN 0902-7513 R8810.

PAPER NO. 4: Jakob Laigaard Jensen: *Dynamic Analysis of a Monopile Model.* ISSN 0902-7513 R8824.

PAPER NO. 5: Rune Brincker & Henrik Dahl: *On the Fictitious Crack Model of Concrete Fracture.* ISSN 0902-7513 R8830.

PAPER NO. 6: Lars Pilegaard Hansen: *Udmattelsesforsøg med St. 50-2, serie 1 - 2 - 3 - 4.* ISSN 0902-7513 R8813.

PAPER NO. 7: Lise Gansted: *Fatigue of Steel: State-of-the-Art Report.* ISSN 0902-7513 R8826.

PAPER NO. 8: P. H. Kirkegaard, I. Enevoldsen, J. D. Sørensen, R. Brincker: *Reliability Analysis of a Mono-Tower Platform.* ISSN 0902-7513 R8839.

PAPER NO. 9: P. H. Kirkegaard, J. D. Sørensen, R. Brincker: *Fatigue Analysis of a Mono-Tower Platform.* ISSN 0902-7513 R8840.

PAPER NO. 10: Jakob Laigaard Jensen: *System Identification1: ARMA Models.* ISSN 0902-7513 R8908.

PAPER NO. 11: Henrik Dahl & Rune Brincker: *Fracture Energy of High-Strength Concrete in Compression.* ISSN 0902-7513 R8919.

PAPER NO. 12: Lise Gansted, Rune Brincker & Lars Pilegaard Hansen: *Numerical Cumulative Damage: The FM-Model.* ISSN 0902-7513 R8920.

PAPER NO. 13: Lise Gansted: *Fatigue of Steel: Deterministic Loading on CT-Specimens.*

PAPER NO. 14: Jakob Laigaard Jensen, Rune Brincker & Anders Rytter: *Identification of Light Damping in Structures.* ISSN 0902-7513 R8928.

PAPER NO. 15: Anders Rytter, Jakob Laigaard Jensen & Lars Pilegaard Hansen: *System Identification from Output Measurements.* ISSN 0902-7513 R8929.

PAPER NO. 16: Jens Peder Ulfkjær: *Brud i beton - State-of-the-Art. 1. del, brudforløb og brudmodeller.* ISSN 0902-7513 R9001.

PAPER NO. 17: Jakob Laigaard Jensen: *Full-Scale Measurements of Offshore Platforms.* ISSN 0902-7513 R9002.

PAPER NO. 18: Jakob Laigaard Jensen, Rune Brincker & Anders Rytter: *Uncertainty of Modal Parameters Estimated by ARMA Models.* ISSN 0902-7513 R9006.

PAPER NO. 19: Rune Brincker: *Crack Tip Parameters for Growing Cracks in Linear Viscoelastic Materials.* ISSN 0902-7513 R9007.

PAPER NO. 20: Rune Brincker, Jakob L. Jensen & Steen Krenk: *Spectral Estimation by the Random Dec Technique.* ISSN 0902-7513 R9008.

FRACTURE AND DYNAMICS PAPERS

PAPER NO. 21: P. H. Kirkegaard, J. D. Sørensen & Rune Brincker: *Optimization of Measurements on Dynamically Sensitive Structures Using a Reliability Approach.* ISSN 0902-7513 R9009.

PAPER NO. 22: Jakob Laigaard Jensen: *System Identification of Offshore Platforms.* ISSN 0902-7513 R9011.

PAPER NO. 23: Janus Lyngbye & Rune Brincker: *Crack Length Detection by Digital Image Processing.* ISSN 0902-7513 R9018.

PAPER NO 24: Jens Peder Ulfkjær, Rune Brinker & Steen Krenk: *Analytical Model for Complete Moment-Rotation Curves of Concrete Beams in bending.* ISSN 0902-7513 R9021.

PAPER NO 25: Leo Thesbjerg: *Active Vibration Control of Civil Engineering Structures under Earthquake Excitation.* ISSN 0902-7513 R9027.

PAPER NO. 26: Rune Brincker, Steen Krenk & Jakob Laigaard Jensen: *Estimation of correlation Functions by the Random Dec Technique.* ISSN 0902-7513 R9028.

PAPER NO. 27: Jakob Laigaard Jensen, Poul Henning Kirkegaard & Rune Brincker: *Model and Wave Load Identification by ARMA Calibration.* ISSN 0902-7513 R9035.

PAPER NO. 28: Rune Brincker, Steen Krenk & Jakob Laigaard Jensen: *Estimation of Correlation Functions by the Random Decrement Technique.* ISSN 0902-7513 R9041.

PAPER NO. 29: Poul Henning Kirkegaard, John D. Sørensen & Rune Brincker: *Optimal Design of Measurement Programs for the Parameter Identification of Dynamic Systems.* ISSN 0902-7513 R9103.

PAPER NO. 30: L. Gansted & N. B. Sørensen: *Introduction to Fatigue and Fracture Mechanics.* ISSN 0902-7513 R9104.

PAPER NO. 31: R. Brincker, A. Rytter & S. Krenk: *Non-Parametric Estimation of Correlation Functions.* ISSN 0902-7513 R9120.

PAPER NO. 32: R. Brincker, P. H. Kirkegaard & A. Rytter: *Identification of System Parameters by the Random Decrement Technique.* ISSN 0902-7513 R9121.

Department of Building Technology and Structural Engineering
The University of Aalborg, Sohngaardsholmsvej 57, DK 9000 Aalborg
Telephone: 45 98 15 85 22 Telefax: 45 98 14 82 43